TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 2 2019

Calculator Assumed
Component Vectors

STUDENT'S NAME ______________________________

DATE: Friday 5th April **TIME:** 50 minutes **MARKS:** 47

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser and notes

---

1. (6 marks)

Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = -\mathbf{i} + 5\mathbf{j}$ and $\mathbf{c} = 5\mathbf{i} - 12\mathbf{j}$, determine:

(a) $\mathbf{a} - 2\mathbf{c}$ [2]

$= (2i + 3j) - (10i - 24j)$ ✓

$= -8i + 27j$ ✓

(b) $|\mathbf{a} - 2\mathbf{c}|$ [2]

$= \sqrt{(8)^2 + (27)^2}$ ✓

$= \sqrt{793}$ ✓

$= 28.16$

(c) $\mathbf{a} \cdot \mathbf{b}$ [2]

$= (2)(-1) + (3)(5)$ ✓

$= 13$ ✓

2. (8 marks)

Given $\mathbf{a} = \mathbf{i} - \mathbf{j}$, $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} + 5\mathbf{j}$, determine:

(a) $\hat{\mathbf{a}}$, the unit vector in the same direction as $\mathbf{a}$ [2]

$|a| = \sqrt{2}$ ✓

$\hat{\underset{\sim}{a}} = \frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$ ✓

(b) The vector $\mathbf{d}$, which is in the opposite direction as $\mathbf{a}$ with double the magnitude of $\mathbf{b}$ [3]

$\underset{\sim}{d} = -\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$ ✓ $|\underset{\sim}{b}| = 5$ ✓

$= \frac{-10}{\sqrt{2}}\underset{\sim}{i} + \frac{10}{\sqrt{2}}j$ ✓

(c) The vector projection of $\mathbf{a}$ onto $\mathbf{c}$ [3]

$\text{Scalar proj} = \frac{(1)(2) + (-1)(5)}{\sqrt{29}}$ ✓

$= \frac{-3}{\sqrt{29}}$ ✓

$\text{vector proj} = \frac{-3}{\sqrt{29}} \times \left(\frac{2}{\sqrt{29}}i + \frac{5}{\sqrt{29}}j\right)$

$= \frac{-6}{29}i - \frac{15}{29}j$ ✓

3. (4 marks)

For the two non-parallel vectors **c** and **d**, determine the values of λ and μ for which:

$$2\lambda\mathbf{c} - \lambda\mathbf{d} = 10\mathbf{c} + 2\mu\mathbf{d} + \mu\mathbf{c}$$

$$2\lambda c - 10c - \mu c = 2\mu d + \lambda d$$

$$c(2\lambda - 10 - \mu) = d(2\mu + \lambda) \quad \checkmark$$

$$2\lambda - 10 - \mu = 0 \qquad 2\mu + \lambda = 0 \quad \checkmark$$

$$\mu = -2 \quad \checkmark$$

$$\lambda = 4 \quad \checkmark$$

4. (5 marks)

If **a** = 7**i** – 3**j**, **b** = μ**i** + 5**j**, determine the possible value(s) of μ given that:

(a) **a** and **b** are parallel vectors. [2]

$$\frac{\mu}{7} = \frac{5}{-3} \quad \checkmark$$

$$\mu = \frac{-35}{3} \quad \checkmark$$

(b) (**a** + **b**) and **b** are perpendicular vectors. [3]

$$(\underset{\sim}{a} + \underset{\sim}{b}) = (7+\mu)\underset{\sim}{i} + 2\underset{\sim}{j} \quad \checkmark$$

$$(7+\mu)(\mu) + (2)(5) = 0 \quad \checkmark$$

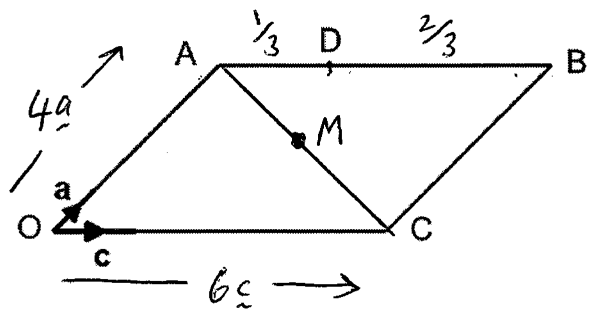
$$\mu = -5$$

or $\quad \checkmark$

$$\mu = -2$$

5. (7 marks)

In the parallelogram OABC below $\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OC} = 6\mathbf{c}$. D is a point on AB such that AB:DB = 1:2.



(a) Express the following in terms of **a** and/or **c**.

(i) $\overrightarrow{AC}$ [1]

$= -4\underset{\sim}{a} + 6\underset{\sim}{c}$

(ii) $\overrightarrow{AD}$ [1]

$= 2\underset{\sim}{c}$

(iii) $\overrightarrow{DC}$ [2]

$= 4\underset{\sim}{c} - 4\underset{\sim}{a}$

(b) M is the midpoint of AC, Express MD in terms of **a** and/or **c**. [3]

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$

$= 2\underset{\sim}{a} + 3\underset{\sim}{c}$

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$= 4\underset{\sim}{a} + 2\underset{\sim}{c}$

$\overrightarrow{MD} = \overrightarrow{OD} - \overrightarrow{OM}$

$= 4\underset{\sim}{a} + 2\underset{\sim}{c} - 2\underset{\sim}{a} - 3\underset{\sim}{c}$

$= 2\underset{\sim}{a} - \underset{\sim}{c}$

6. (6 marks)

The work done, in joules, by a force **F** Newtons in changing the displacement of an object, **s** metres is given by the scalar product of **F** and **s**.

(a) Calculate the work done by a force <15, 22> N in moving an object <3, 2>m. [1]

$W = (15)(3) + (22)(2)$

$= 89\ J$

(b) Calculate the work done by a force of 25 N that moves an object 6 m if:

(i) The force acts parallel to the direction of the movement. [1]

$= 25 \times 6 \times \cos 0$

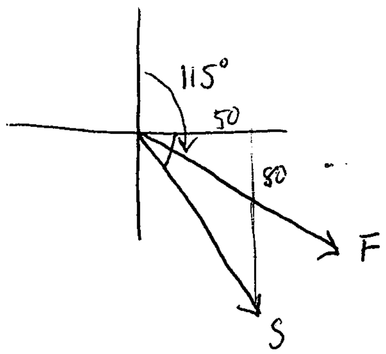
$= 150\ J$

(ii) The force acts perpendicular to the direction of movement. [1]

$= 25 \times 6 \times \cos 90$

$= 0\ J$

(c) The work done by a force in moving an object <50, -80> cm is 590 joules. If the force acts on a bearing of 115°, determine the magnitude of the force. [3]



$\tan\theta = \frac{80}{50}$

$\theta = 58°$

Angle between F & S

$= 58° - 25°$

$= 33°$

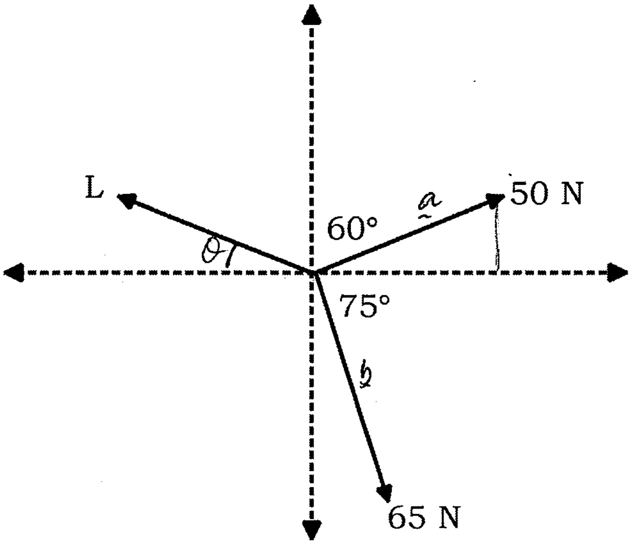
$590 = |\underset{\sim}{F}| \times \sqrt{(50)^2 + (80)^2} \times \cos 33$

$|\underset{\sim}{F}| = 745.7\ J$

7. (5 marks)

Luke, Yoda and Leia are trying to move an X-Wing fighter.

The diagram below shows the magnitude and direction of the three forces exerted by the three people.



Determine the magnitude and direction of the force that Leia must exert if the three forces are in equilibrium.

$\underset{\sim}{a} = (50\cos 30)\underset{\sim}{i} + (50\sin 30)\underset{\sim}{j}$ ✓

$\underset{\sim}{b} = (65\cos 75)\underset{\sim}{i} - (65\sin 75)\underset{\sim}{j}$

$r = 60.12\underset{\sim}{i} - 37.79\underset{\sim}{j}$ ✓

$\therefore L = -r$

$L = -34.14\underset{\sim}{i} + 37.79\underset{\sim}{j}$ ✓

$|\underset{\sim}{L}| = \sqrt{(34.14)^2 + (37.79)^2}$

$= 50.93N$ ✓

$\theta = \tan^{-1}\left(\frac{37.79}{34.14}\right)$

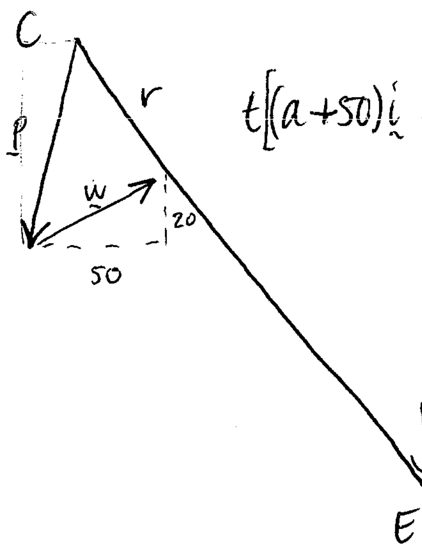
$\theta = 47.90°$ ✓

$\therefore$ 50.93N @ 47.90° to the horizontal

9. (6 marks)

The distance between two towns, Charleton and Edensville is given by $\begin{pmatrix} 162 \\ -2115 \end{pmatrix}$ km. An aircraft is to be flown directly from Charleton to Edenville. This particular aircraft can maintain a steady speed of 257 km/h in still air but for the duration of this flight there is a wind blowing with a constant velocity of $\begin{pmatrix} 50 \\ 20 \end{pmatrix}$ km/h.

Determine, in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ km/h, the velocity vector that the pilot must set to fly directly from Charleton to Edensville and determine the time that journey takes.



$$t\,r = CE$$

$$t\left[(a+50)\underset{\sim}{i} + (b+20)\underset{\sim}{j}\right] = 162\underset{\sim}{i} - 2115\underset{\sim}{j}$$

$$t(a+50) = 162$$

$$t(b+20) = -2115$$

$$\sqrt{a^2+b^2} = 257$$